SWITCH REDUCTION FOR BIDIRECTIONAL TELEPHONE SYSTEMS

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A method is described which, for any n, generates a rearrangeably nonblocking network for a bidirectional telephone system containing $2n$ subscribers. The number of switches is always $n^2 + 2n - 1$, which is the minimum possible for two-stage networks of this type. This generalises earlier work of Newbury and Raby.

Introduction: In two recent papers,[2,3] Newbury and Raby have considered the use of two-stage switching arrangements in bidirectional telephone systems. In the second paper[3] an example is given of an arrangement for eight subscribers which uses 23 switches, and the claim is made that exhaustive computer searches have shown that this is the minimum possible for a network of this type. Newbury and Raby assert that two-stage networks have the potential to give the minimum number of switches for bidirectional telephone systems containing less than 14 subscribers, and so finding minimal switching arrangements for 10 and 12 subscribers remains a problem of interest.

In this letter we give a method which, for any value of n, can be used to construct a rearrangeably nonblocking switching arrangement for $2n$ subscribers having $n^2 + 2n - 1$ switches. Moreover, we also give a theoretical result showing this to be the minimum possible for any value of n.

Notation and definitions: We are concerned here with two-stage bidirectional telephone systems. To be more specific, we are concerned with the situation where $2n$ subscribers are connected using n crosswires, where each subscriber is connected by a switch to some (or all) of the crosswires. For convenience let the $2n$ subscribers be labelled $\{P_1, P_2, \ldots, P_{2n}\}$, label the crosswires $\{x_1, x_2, \ldots, x_n\}$ and let S be the set of pairs (P_i, x_j) of subscribers and crosswires connected by a switch; i.e. subscriber P_i is connected by a switch to crosswire x_j if and only if $(P_i, x_j) \in S$. We call such an arrangement a two-stage bidirectional switching network.

For the network to be rearrangeably nonblocking (sometimes just called 'rearrangeable') we require the following property. For every partition of the $2n$ subscribers into n pairs, $\{P_{f(1)}, P_{f(2)}\}, \{P_{f(3)}, P_{f(4)}\}, \ldots, \{P_{f(2n-1)}, P_{f(2n)}\}$, say (where f is a permutation of $\{1, 2, \ldots, 2n\}$), there exists an ordering of the n crosswires $\{x_{g(1)}, x_{g(2)}, \ldots, x_{g(n)}\}$, say (where g is a permutation of $\{1, 2, \ldots, n\}$), such that $(P_{f(2i-1)}, x_{g(i)}) \in S$ and $(P_{f(2i)}, x_{g(i)}) \in S$ for every i $(1 \leq i \leq n)$; i.e. for every possible set of n telephone calls each call can be assigned to a unique crosswire having switches in the appropriate two places. However, if some calls cease and the corresponding subscribers need to be reconnected in a different way, some rearrangement of existing calls on to different crosswires may be necessary (hence the term 'rearrangeably' nonblocking).

If a network satisfies the above property then we call it a rearrangeable nonblocking two-stage bidirectional switching network for $2n$ subscribers, or an R2BSN(n) for short. For every positive integer n we are concerned with finding an R2BSN(n) with the minimum number of switches, i.e. with the minimum size for S. An example of an R2BSN(4) having the minimal number of switches (23) is given in Fig. 1.

Construction method: Suppose n is even, i.e. let $n = 2m$; we now construct an R2BSN(n). To do this we just list the elements of S.

If $1 \leq i \leq m$ then $(P_{2i-1}, x_i) \in S$, $(P_{2i}, x_i) \in S$ and $(P_j, x_i) \in S$ for every j satisfying $2m + 1 \leq j \leq 4m$. If $m + 1 \leq i \leq 2m$ then $(P_{2i-1}, x_i) \in S$, $(P_{2i}, x_i) \in S$ and $(P_j, x_i) \in S$ for every j satisfying $1 \leq j \leq 2m$, with the exception that (P_{4m}, x_{2m}) is *not* in S.

The fact that this is an R2BSN($2m$) can be verified by using Hall's marriage theorem.[1] Using this theorem we need only show that for any set of s disjoint pairs of subscribers $(1 \leq s \leq n)$ there are at least s crosswires joining one or more of the pairs. Note that for this network the number of switches is $2m(2m + 2) - 1 = n^2 + 2n - 1$.

If we now take the above network and delete subscribers P_{2m} and P_{4m} and also delete crosswire x_m, we obtain an R2BSN($2m - 1$) having $(4m^2 + 4m - 1) - (4m + 1) = (2m - 1)^2 + 2(2m - 1) - 1$ switches. Thus we have shown how to construct an R2BSN(n) having $n^2 + 2n - 1$ switches for every value of n. We now see that this is the smallest number of switches that such a network can have.

Before proceeding note that the configuration given in Newbury's paper[3] is an example of the above construction method for the case $n = 4$. However, the example given in Fig. 1 is not obtainable from the above technique.

Theoretical results: Using combinatorial arguments we are able to obtain the following lemmas:

Lemma 1: In an R2BSN(n) every crosswire must be connected by a switch to at least $n + 1$ subscribers.

Lemma 2: In an R2BSN(n) at most one crosswire is connected by a switch to precisely $n + 1$ subscribers.

Combining these two lemmas we immediately have the following result:

Theorem 3: An R2BSN(n) contains at least $n^2 + 2n - 1$ switches. By this theorem we can immediately see that the construction method described above always gives an R2BSN(n) with the minimum possible number of switches.

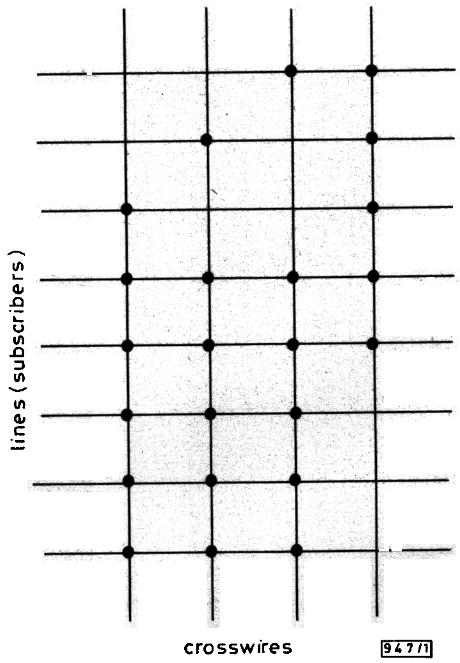

Fig. 1 *An R2BSN(4) with minimal switches*

Concluding remarks: We have exhibited the existence of a switching network with the desired properties and the minimum number of switches for every possible (even) number of subscribers. Although the configurations constructed here are minimal they are certainly not unique; for example, compare the pattern given in Fig. 1 with the example in Newbury's paper.[3] This and other questions will be discussed in a future paper, in which full proofs will be given for the results described in this letter.

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