

ON DIVISIONS AND DECOMPOSITIONS OF 1-DESIGNS

BY

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ABSTRACT of "On Divisions and Decompositions of 1-designs"

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A Point Division of a $1-(v,k,r)$ structure \underline{S} is a partition of the points of \underline{S} into classes such that the number of blocks through two points depends only on the classes to which they belong. This generalises the notion of a Group Divisible (GD) design and a number of results are obtained for Point Divisions of 1-designs which have well-known results for GD designs as corollaries.

Point Divisions are also closely linked to tactical divisions; in fact a tactical division of a 1-design is a tactical decomposition whose point classes form a certain special type of Point Division. Using this fact we obtain simple proofs of certain results on designs admitting tactical divisions. We also examine 2-designs whose duals admit Point Divisions, and show that this is equivalent to considering 2-designs having intersection number $k-r+\lambda$.

Using results obtained for Point Divisions of 1-designs, we go on to establish new results on GD designs, in particular we derive information about the duals of GD designs, and the properties of GD designs having certain special dual properties. We also obtain necessary and sufficient conditions for a symmetric GD design to have a GD dual.

Finally we give a general recursive method of construction for 1-structures admitting Point Divisions having constant class size. This method is used to construct both GD and 2-designs, and we use it to obtain two infinite families of strongly divisible 2-designs. One of these infinite families consists of quasi-residual designs, and we show that they are in fact residual designs. This establishes the existence of an infinite family of symmetric 2-designs.

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